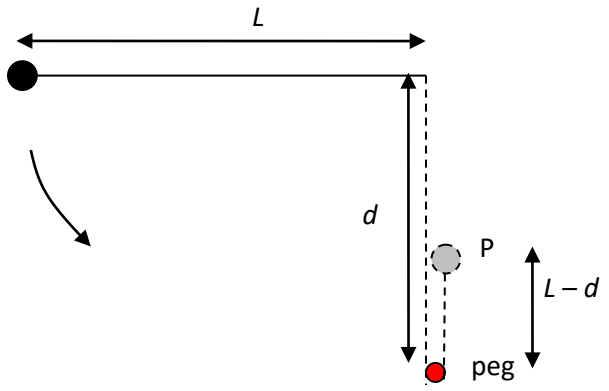
Teacher notes
Topic A

A pendulum meets a peg

The string of a pendulum of length *L* is held horizontal. A peg is a distance *d* below the support point of the pendulum. The pendulum is released. What is the smallest *d* so that the tension in the string at position P is not zero?



After hitting the peg, the pendulum bob will move along a circular path of radius $L-d$. At position P the bob is a distance $d-(L-d)=2d-L$ below the support point. By energy conservation

$$mg(2d-L)=\frac{1}{2}mv^2 \Rightarrow v^2=2g(2d-L) \text{ at P.}$$

The net force on the bob at P is $T+mg$ and so $T+mg=\frac{mv^2}{(L-d)}$. Then

$$T=\frac{mv^2}{(L-d)}-mg$$
$$T=\frac{2mg(2d-L)}{L-d}-mg\frac{L-d}{L-d}$$
$$T=\frac{mg}{L-d}(5d-3L)$$
$$T>0$$
$$5d>3L$$
$$d>\frac{3L}{5}$$